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Spacelike and time dependent branes from DBI

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Abstract: Spacelike branes are new time-dependent systems to explore and it has been observed that related supergravity solutions can be obtained by analytically continuing known D-brane solutions. Here we show that analytic continuation of known solutions of the Dirac-Born-Infeld equations also lead to interesting analogs of time dependent gravity solutions. Properties of these new solutions, which are similar to the Witten bubble of nothing and S-branes, are discussed. We comment on how these new bubble solutions seem relevant to the tachyon condensation process of non-BPS branes, and remark on their application to cosmological scenarios. Unstable brane configurations which resemble S-brane type solutions are also discussed.

Keywords: D-branes, Brane Dynamics in Gauge Theories, Tachyon Condensation.

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1. Introduction

Time dependent backgrounds have arisen in the study of tachyon condensation [1, 2], as well as gravitational backgrounds related to de Sitter space [3, 4] and inflationary scenarios [5]–[10]. A class of time dependent solutions called S-branes [11, 12] has been found, and related supergravity [13]–[16] solutions have already been explored. In this paper we approach the topic of time dependent systems by examining solutions of the Dirac-Born-Infeld action. We find similar braney objects in the sense that the induced metric on these solutions is similar to the metric of the known gravity solutions.

In section 2 we give a quick review of the DBI action and its string-like excitation solution [17, 18]. We discuss how this static brane and anti-brane configuration resembles charged wormhole solutions from black hole theory. In section 3 we discuss new DBI solutions which come from analytically continuing the string-like excitation solutions. These brane configurations are similar to Witten bubbles [19] and play a role in brane and anti-brane annihilation. An important difference between the new bubble solutions presented here and the previous gravity solution, is that a Witten bubble "eats up" spacetime while the bubble solution in this paper is a safer process which does not effect the bulk of spacetime. In our case we have a time dependent brane and anti-brane configuration in flat space.

Section 4 introduces solutions which appear similar to S-brane solutions and which also arise from analytically continuing the string-like excitation solution. These solutions are presented and discussed although their interpretation is more difficult since they are embedded in spaces with two times.

2. Review of DBI

In this section we will give a quick review of the DBI action and its equations of motion. We review the string-like excitation [17, 18] of a brane and give an explicit embedding of the solution. In addition we give the induced metric on this brane and anti-brane configuration which is a type of static wormhole. Discussion of the various p-branes solutions is separated into three categories $p > 3$, $p = 3$ and $p < 3$ which exhibit qualitatively different behaviors.

2.1 Equations of motion

The Dirac-Born-Infeld action

$$S = T_p \int d^{p+1}x \sqrt{-\det(G_{MN}\partial_a Z^M \partial_b Z^N + F_{ab})} \tag{2.1}$$

is the worldvolume action describing the shape and abelian gauge field of a Dp-brane embedded in a $D+1$ dimensional space, M_2. The metric on M_2 is G_{MN} and the indices M, N range from 0 to D. The coordinates $x^a,\ a = 0, \ldots, p$ are the coordinates for the brane worldvolume while the functions

$$Z^M : \ M_1 \to M_2 \,, \qquad \forall M \tag{2.2}$$

are the embedding coordinates of the worldvolume manifold M_1 into the embedding space M_2. For simplicity we will restrict ourselves to the case where the embedding space is flat pseudo-euclidean spacetime $M_2 = R^{(p,q)}$. We will also take the cartesian coordinate representation of M_2 so the Christoffel symbols vanish.

This action provides the low energy effective description of open string dynamics provided we take the usual light brane or decoupling limit [20] which neglects backreaction effects of the brane on the target space. The shape of the brane is obtained by varying the action

$$\delta_Z S = -\frac{T_p}{2} \int d^{p+1}x \ \sqrt{-(g+F)}(g+F)^{ab}\delta(G_{MN}\partial_a Z^M \partial_b Z^N) = 0 \,. \tag{2.3}$$

The equations of motion for the DBI action are

$$\partial_a \left(\sqrt{-(g+F)}(g+F)^{ab}_S \partial_b Z^M \right) = 0 \,, \qquad \forall \ M \tag{2.4}$$

$$\partial_b \left(\sqrt{-(g+F)}(g+F)^{ba}_A \right) = 0 \,, \qquad \forall \ a \tag{2.5}$$

where the second equation comes from varying the action with respect to the gauge potential. In the above expressions $(g+F)^{ab}_{A/S}$ is the (anti-)symmetric part of the inverse of the matrix $g+F$ which is generally different from the sum of inverses. It is known that this system of equations can also be obtained from a pure Born-Infeld system with just a field strength.

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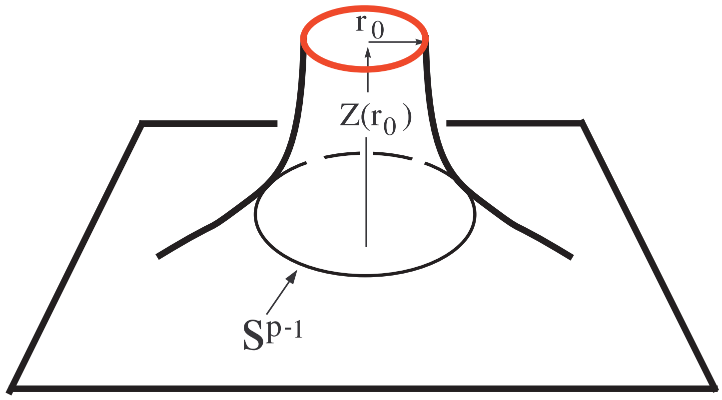

Figure 1: Static volcano solution of a Dp brane.

2.2 String-like brane intersections

The known string type excitation of the DBI action can be presented in the following parametrization

$$Z^0 = t \tag{2.6}$$
$$Z^1 = r\cos\theta \tag{2.7}$$
$$Z^i = r\sin\theta n^i \qquad i = 2,\ldots,p \tag{2.8}$$
$$Z^{p+1} = \int \frac{B}{\sqrt{r^{2p-2} - r_0^{2p-2}}}dr\,. \tag{2.9}$$

The rest of the Z coordinates are constant functions which will not play a dynamical role in the discussion and can be set to zero. The above parametrization satisfies the constraints $\sum_{a=1}^{p}(Z^a)^2 = r^2$ and $\sum_i (n^i)^2 = 1$. The vector $\vec{n}$ is a coordinate free way to specify the angular coordinates of the brane worldvolume; an example of an explicit parametrization is $n^i = (\cos\phi_1, \sin\phi_1\cos\phi_2, \sin\phi_1\sin\phi_2\cos\phi_3, \ldots)$. The first $p+1$ coordinates are coordinates of the brane worldvolume and the Z^{p+1} coordinate is the one dimensional string-like excitation direction of the brane. For the cases $p \geq 3$ the above solution asymptotes to $Z^{p+1} = 0$ as the radius r approaches infinity. Also, even though the derivative $\partial_r Z^{p+1}$ goes to infinity at $r = r_0$, the value of Z^{p+1} approaches a finite value $Z^{p+1}(r_0)$ when $r_0 \neq 0$. Examining this string-like excitation of the brane, illustrated in figure 1, we find that it has the appearance of a volcano where the radius of the volcano's mouth is r_0 and the height of the volcano is $Z^{p+1}(r_0)$.

The embedding space is Minkowski with metric $G_{MN} = \eta_{MN} = (-1, 1, 1, \ldots, 1)$ where $M, N = 0, \ldots, D$ so the induced metric on this brane is

$$ds^2_{\text{volcano}} = \eta_{\mu\nu}dZ^\mu dZ^\nu = -dt^2 + \frac{1 + A^2/r^{2p-2}}{1-(r_0/r)^{2p-2}}dr^2 + r^2(d\theta^2 + \sin^2\theta d\Omega^2_{p-2}) \tag{2.10}$$
$$A^2 \equiv B^2 - r_0^{2p-2}\,. \tag{2.11}$$

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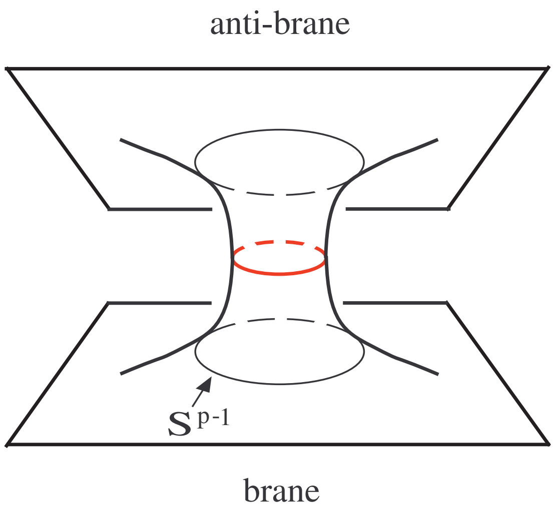

Figure 2: Two volcanos, representing a brane and anti-brane, are joined to form a wormhole.

The variable A plays a role in determining the field strength of this solution

$$F_{tr} = \frac{A}{\sqrt{r^{2p-2} - r_0^{2p-2}}} \tag{2.12}$$

which will play an interesting role later in our discussion. To insure that the electric field is real valued, the parameters must satisfy $B^2 \geq r_0^{2p-2}$. We note here that the field strength diverges at $r = r_0$ and there is no source of charge along the brane.

If we glue two such volcano solutions together at the radius $r = r_0$ then this is a static configuration, shown in figure 2, representing a brane and anti-brane configuration connected by a wormhole throat. An explicit way to do this is to take two copies of the above parametrization but with

$$Z^{p+1} = 2Z^{p+1}(r_0) - \int \frac{B}{\sqrt{r^{2p-2} - r_0^{2p-2}}} dr \tag{2.13}$$

for the second copy. We note that this configuration is similar to the Einstein-Rosen bridge where the radius r_0 corresponds to the horizon and the brane and the anti-brane correspond to the two asymptotically Minkowski regions. This solution is smooth and does not have a singular region of infinite curvature. It is like a charged black hole where the constant A is an electric charge parameter, B the mass parameter and r_0 tells us how far away we are from the BPS configuration.

Several limits are of interest. The first is the limit, $A^2 = B^2$ where the electric charge is equal to the mass. In this limit r_0 is zero and the string excitation stretches to infinity. This BPS configuration is interpreted as a half-infinite string ending on a brane with the end of the open string acting as an electric charge source for the gauge field on the brane.

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There is also the interesting non-BPS limit $A = 0$ with no electric charge. This solution

$$ds^2_{\text{no charge volcano}} = -dt^2 + \frac{dr^2}{1 - (r_0/r)^{2p-2}} + r^2(d\theta^2 + \sin^2\theta d\Omega^2_{p-2}) \tag{2.14}$$

is similar to a Schwarzschild black hole. However whereas the Schwarzschild black hole can be extended past the horizon, the above parametrization does not reach into the singular region. Also the time direction of this volcano solution is independent of the radius, so dynamics on this brane will not be the same as on a Schwarzschild spacetime [17].

Because the volcano solutions are not supersymmetric they have also been mentioned as being related to possible decay modes for brane and anti-brane annihilation. However since these solutions are time independent, it is interesting to obtain explicit time dependent annihilation processes. In the next section we will discuss new time dependent solutions and their relevance to brane and anti-brane annihilation.

2.3 Low dimensional volcano solutions

For the case of $D3$ branes, there is a similar configuration to the one mentioned above where the role of electric and magnetic charges is switched. In this case we turn on a magnetic field in the radial direction

$$F_{\theta\phi} = \sqrt{B^2 - r_0^4}\sin\theta \quad \leftrightarrow \quad B_r = \sqrt{\frac{1-(r_0/r)^4}{1+A^2/r^4}}\frac{A}{r^2} \tag{2.15}$$

in which case the excitation is not a fundamental string but a D-string.

For the case of two-branes, the excitation does not asymptote to a fixed value in the large radius r limit. The integral expression for Z^3 can be solved explicitly in this case and has logarithmic behaviour

$$r \to \infty \qquad \Rightarrow \qquad Z^3 = B\ln\frac{1}{2}\left|\sqrt{r^2} + \sqrt{r^2 - r_0^2}\right| \to \ln r\,. \tag{2.16}$$

This logarithmic behavior, shown in figure 3, is expected for a system of two-branes instead of two-branes connected by a string, and indicates that we are capturing non-perturbative string coupling effects.

Finally, for the case $p = 1$ we obtain string junction configurations which are continuous but not necessarily smooth. An example of a three string junction is shown in figure 4.

3. Bubble solution

Given a solution to the DBI equations, we can generate new solutions by analytic continuation. Here we will use this procedure to generate time dependent solutions which seem to play a role in brane and anti-brane annihilation. Specifically we analytically continue the volcano solution of the previous section and obtain an analogue of Witten's bubble of nothing. This is a DBI approach to brane and anti-brane annihilation with some similarities to the supergravity approach discussed by Fabinger and Horava [21]. They studied a non-supersymmetric $E_8 \times \overline{E_8}$ compactification of M theory on S_1/Z_2

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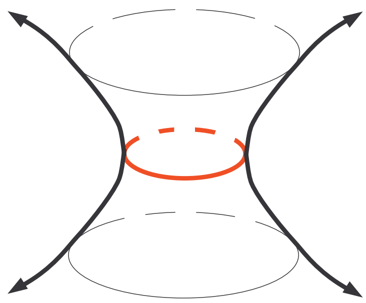

Figure 3: The solution for two branes is logarithmic and does not asymptote to a fixed value.

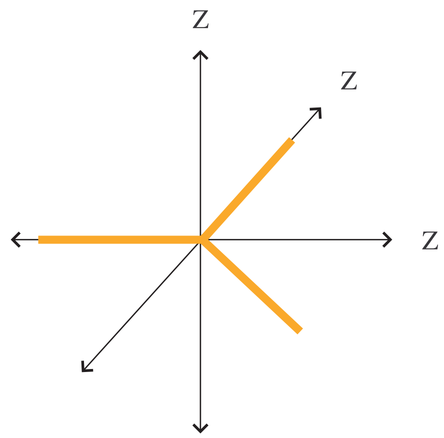

Figure 4: Three string junction with spatial directions labeled schematically by Z.

and proposed that spacetime could semiclassically annihilate due to wormholes nucleating between the two E_8 boundaries. The role of bubble type solutions has also been discussed by K. Hashimoto in [22] especially from the viewpoint of time evolution of the tachyon field.

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3.1 Bubble embedding

Motivated by the Witten bubble solution, we examine the following analytic continuation of the embedding in the previous section. If we make the analytic continuations $t \to i\chi$ and $\theta \to \pi/2 - i\tau$ then the embedding functions become

$$Z^0 = \chi \tag{3.1}$$

$$Z^1 = r \sinh \tau \tag{3.2}$$

$$Z^i = r \cosh \tau n^i \,, \qquad i = 2, \ldots, p \tag{3.3}$$

$$Z^{p+1} = \int \frac{B}{\sqrt{r^{2p-2} - r_0^{2p-2}}} dr \,. \tag{3.4}$$

The coordinates Z^M for $M = 0$ to p are along the brane worldvolume while Z^{p+1} is again the direction of the excitation of the configuration. The embedding functions are all real because we take this solution to be embedded into flat Minkowski space with metric $\eta_{MN} = (1, -1, 1, \ldots, 1)$. The induced metric on this brane is

$$ds^2_{\text{bubble}} = d\chi^2 + \frac{1 - A^2/r^{2p-2}}{1 - (r_0/r)^{2p-2}} dr^2 + r^2(-d\tau^2 + \cosh^2 \tau d\Omega^2_{p-2}) \tag{3.5}$$

$$A^2 \equiv r_0^{2p-2} - B^2 \,. \tag{3.6}$$

Here we impose the restriction $r_0^{2p-2} \geq B^2$ which is the reverse inequality relative to the volcano solution; the well-defined bubble solutions are analytic continuations of volcano solutions with imaginary field strength. This solution has an expanding bubble situated at the radius $r = r_0$ as will be discussed shortly. We also note that because [23] the embedding space of this bubble metric has only one time, this solution is globally hyperbolic which is just a way of saying that the spacetime has a Cauchy surface. Having a target space with one embedding time effectively means that each constant time slice is a Cauchy surface.

One can ask why we did not drop the coordinate χ from the solution and interpret this configuration as a lower dimensional $p-1$ brane. Although the coordinate χ plays no role in the dynamics of the membrane embedding, one can not simply drop this coordinate in general. The reason why is that the general solution has a magnetic field so dropping χ is equivalent to the limiting case of no field strength. Whereas the solution of the previous section had a radial electric field, this solution comes with a magnetic field in the "angular directions" but which decreases radially

$$F_{\chi r} = \frac{A}{\sqrt{r^{2p-2} - r_0^{2p-2}}} \,. \tag{3.7}$$

Hence, although this solution has a string-like excitation, it can not be interpreted as either a fundamental string or a D-string.

3.2 Properties

To obtain a clearer picture of the shape of this solution, see also figure 5, consider the constant χ and $Z^1 = \tau = 0$ slice of the Dp brane bubble solution. We find that this slice is the same as a constant time slice of a D$(p-1)$ volcano solution

$$ds^2_{\text{bubble } \tau=0} = \frac{1 - A^2/r^{2p-2}}{1 - (r_0/r)^{2p-2}} dr^2 + r^2 d\Omega^2_{p-2} \,. \tag{3.8}$$

Therefore if we glue together two bubble solutions, we can also interpret this configuration as a smooth brane and anti-brane system. Just as in the case of the volcano solution we still have the same change in orientation when we cross the horizon r_0. This solution therefore is a time-dependent solution related to brane and anti-brane annihilation.

When there are no gauge fields, it is safe to drop the χ coordinate which does not play a role in this solution. In this case it is possible to interpret the remaining solution as a lower dimensional $p-1$ brane and it is easy to see why this bubble solution is time dependent as

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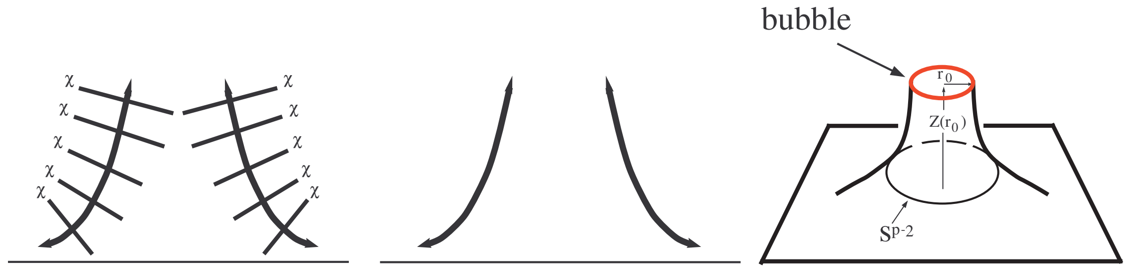

Figure 5: From left to right we have: profile of bubble solution with transverse χ direction included, profile with χ suppressed, full view of solution with χ suppressed.

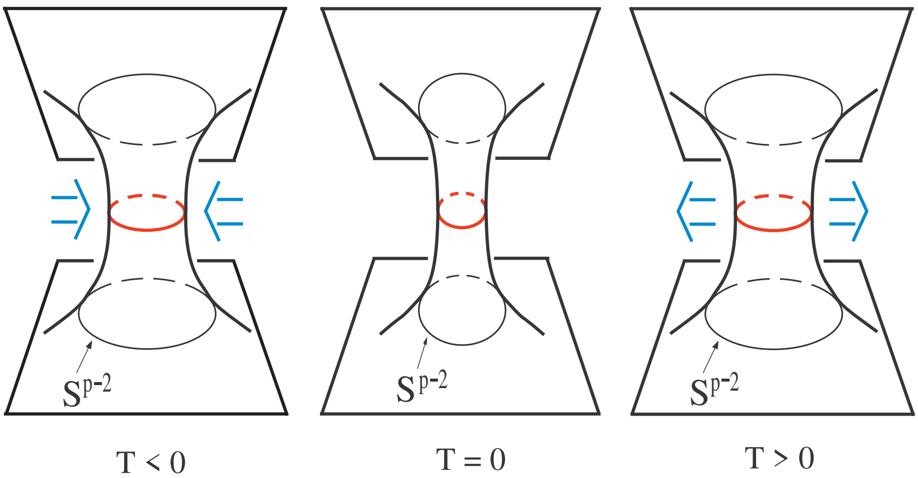

Figure 6: Bubble solution for a p-brane configuration at three different times and with the coordinate χ suppressed. The bubble is the minimum volume S^{p-2}. The bubble itself undergoes contraction and then expansion, while the volume of the brane expands and then contracts.

compared to the volcano solution. Comparing the p brane volcano solution and the $p-1$ dimensional bubble solution, we see that they have identical excitation profiles. If the p brane volcano solution is just massive enough to support this excitation, then the $p-1$ brane can not be massive enough to keep this excitation static and therefore the bubble expands. In fact as shown in figure 6, if we consider the full solution it shows a brane and anti-brane rapidly coming in from infinity, slowing to a stop when they become nearly parallel, and then annihilating. The energy of the branes is transferred into accelerating the bubble wall. This solution has the relevant boundary conditions to describe parallel brane and anti-brane annihilation with no field strength. This expanding bubble solution derives its name from the fact that there is a minimal volume sphere from the worldvolume perspective.

In general, examining the brane with the above worldvolume coordinates leads to unusual behavior for the light cones directions. For example if we study the radial null geodesics, we find that $(dr/d\tau)^2 = r^2[1-(r_0/r)^{2p-2}]$ which does not go to a fixed value when r goes to infinity. On the other hand, we expect the spacetime to be asymptotically Minkowski since the brane flattens out to $Z^{p+1} = 0$. Also it is unusual that there is only a magnetic field in these coordinates since the spacetime is time dependent. This is a sign

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that to better understand the dynamics, especially at infinity, we should express the system in another set of coordinates to make contact with our usual intuition.

One can choose the following coordinates to express the solution

$$W = r\cosh\tau\,, \qquad T = r\sinh\tau \tag{3.9}$$

$$ds^2_{\text{bubble}} = d\chi^2 - dT^2 + dW^2 + \frac{1 - A^2/r_0^{2p-2}}{(\sqrt{W^2 - T^2}/r_0)^{2p-2} - 1}\frac{(WdW - TdT)^2}{W^2 - T^2} + W^2 d\Omega^2_{p-2}\,. \tag{3.10}$$

This is a natural choice of coordinates since the new time coordinate along the brane, T, is the time coordinate of the embedding space. In these cartesian coordinates the spacetime becomes asymptotically Minkowski with the expected light cone property $(dW/dT)^2 \to 1$ as W goes to infinity. We see then that the essence of the problem was that there was a discrepancy between worldvolume time and embedding time. In these coordinates we have both an electric and a magnetic field

$$F_{\chi T} = -\frac{T}{\sqrt{W^2 - T^2}}\frac{A}{[(\sqrt{W^2 - T^2})^{2p-2} - r_0^{2p-2}]^{1/2}} \tag{3.11}$$

$$F_{\chi W} = \frac{W}{\sqrt{W^2 - T^2}}\frac{A}{[(\sqrt{W^2 - T^2})^{2p-2} - r_0^{2p-2}]^{1/2}}\,. \tag{3.12}$$

From the magnitudes of the field strength it appears that this solution looks like a time dependent electric dipole and a magnetic charge at large W distances. However the field configurations have the electric field pointed along the χ direction and the magnetic field is in the angular directions. At time $T = 0$ there is only a magnetic field and no electric field. If we stay at a fixed position W which is far from the bubble, r_0, then we see the electric field decreasing and the magnetic field increasing. Another way to say this is that an initial magnetic field gets pushed in the direction χ, along which the brane is smeared, in the form of an electric field. This process is illustrated in figure 7. In this case the brane and anti-brane annihilate in all spatial directions except χ which remains unchanged for all times. This direction can not be interpreted as a fundamental string since there is no electric field at $T = 0$. It would be interesting however to see if there are similar situations where χ can be identified as a remnant of the annihilation process.

The special case when r_0 approaches zero is also interesting. As r_0 goes to zero, the excitation becomes narrower while the distance between the brane and anti-brane increases; the width of the throat increases in time just like for all the other bubble solutions. However the case $r_0 = 0$ is qualitatively different and has no excitation at all. We are describing only a flat brane or anti-brane. A comparison of the solutions is given in figure 8. The configuration is flat because the constraint $A^2 + B^2 = r_0^{2p-2}$, implies if $r_0 = 0$ then A and B are zero. In this case we can drop the χ direction and the solution is

$$Z^1 = r\sinh\tau \tag{3.13}$$

$$Z^i = r\cosh\tau n^i\,, \qquad i = 2, \ldots, p \tag{3.14}$$

$$ds^2_{\text{limit}} = dr^2 + r^2(-d\tau^2 + \cosh^2\tau d\Omega^2_{p-2}) \tag{3.15}$$

which is Minkowski space in unusual coordinates. Setting r constant, which is de Sitter space itself, is not a solution.

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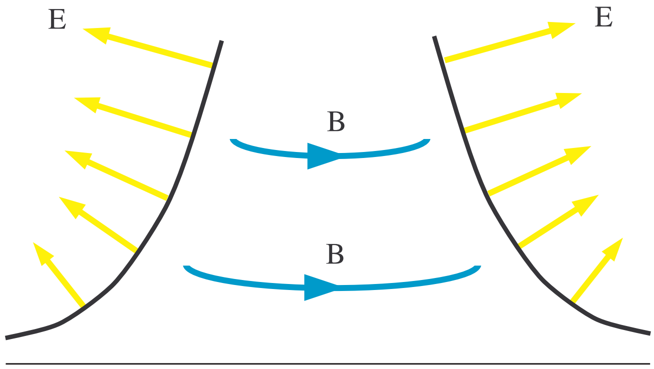

Figure 7: Side profile of a bubble solution with the electric field along the χ direction represented by arrows.

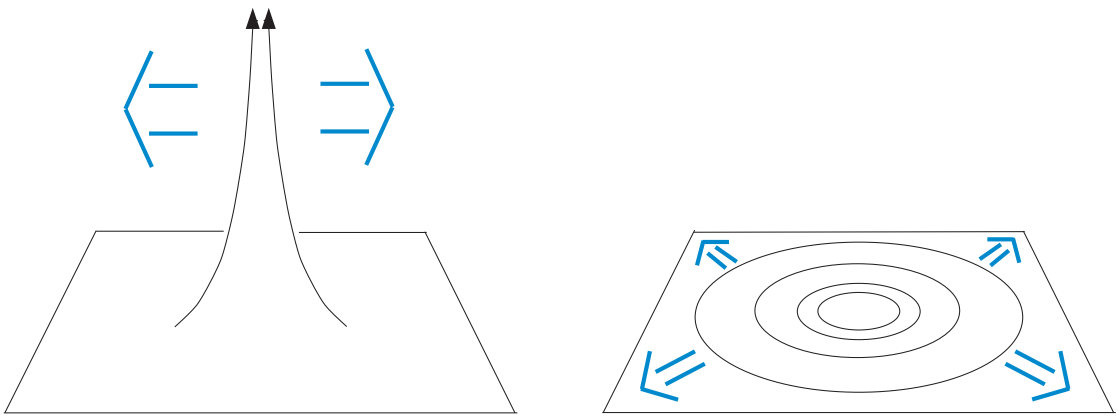

Figure 8: Time evolution for the case of small bubble size compared with zero bubble size.

3.3 Comparison to bubble of nothing

Here we further examine the special case with no field strength. The induced metric on this non-BPS solution is

$$ds^2_{\text{bubble}} = d\chi^2 + \frac{dr^2}{1-(r_0/r)^{2p-2}} + r^2(-d\tau^2 + \cosh^2\tau\, d\Omega^2_{p-2}) \tag{3.16}$$

where for comparison purposes we have reverted back to the spherical coordinate system in eq. (3.1). This is quite similar to the Witten bubble metric

$$ds^2_{\text{WB}} = \left[1-\left(\frac{r_0}{r}\right)^{D-2}\right] d\chi^2 + \frac{dr^2}{1-(r_0/r)^{D-2}} + r^2(-d\tau^2 + \cosh^2\tau\, d\Omega^2_{D-2}) \tag{3.17}$$

which is a vacuum gravity solution for $D+1$ spacetime dimensions with $r \geq r_0$. To make the Witten bubble metric regular at $r = r_0$, we must impose the periodicity condition $\chi = \chi + 4\pi r_0/(D-2)$. Asymptotically the Witten bubble is flat space times a circle whose radius is proportional to the bubble size. Although this periodicity condition is apparently not required for the DBI solution (since χ is independent of the other coordinates),

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actually the periodicity condition is reflected in the fact that we had to join together the two halves of a bubble solution at r_0 to form a smooth wormhole. In other words the distance between the brane and anti-brane is fixed by the minimum size bubble due to the tension of the branes; the only way to change this relationship is by adding charge to the branes.

Although these two solutions are similar, there is never a case where they exactly coincide even if we neglect the χ direction. Setting $D = p$ we find that the DBI solution changes much more quickly in the radial direction than the Witten bubble. One can view a slice of the Witten bubble solution in $D = 2p$ as being exactly the bubble solution. In this case the time coordinate is embedded correctly so dynamics on the brane configuration really are the same as that of a slice of the Witten bubble. Comparing the whole solutions, however, the new bubble solution is closer to a Z_2 projection [21] along the circle direction of the Witten bubble. It is interesting that the DBI solutions are similar only to the Witten bubbles in odd spacetime dimensions. One wonders if this is a reflection of the different causal structures of branes of even versus odd dimension [24]. If so then it suggests the existence of new DBI solutions.

The Witten bubble has also been called a bubble of nothing since there is a "hole" in space due to the fact that we always stay in the region $r \geq r_0$; naively being inside the hole would give a spacetime with three timelike directions. One might ask if spacetime becomes singular or if there is a boundary at r_0. Given that the new bubble in eq. (3.5) is smooth and boundary free, and given the similarities of the two bubble solutions, it should not be surprising that the Witten bubble is smooth and boundary free. To better understand the spacetime near r_0 and see that there is no singularity there, it is convenient to make the coordinate transformation $y = \sqrt{1-(r_0/r)^{D-2}}$. This region is undergoing inflation in the radial direction as can be seen by Taylor expanding the metric

$$ds^2_{WB} \approx r_0^2 \left[\left(1 + \frac{2}{D-2} y^2\right)(-d\tau^2 + \cosh^2 \tau d\Omega^2_{D-2}) + \left(\frac{2}{D-2}\right)^2 \left(1 + 2\frac{D-1}{D-2} y^2\right) dy^2 \right]. \tag{3.18}$$

So near the bubble, and actually at all fixed radius points, the spacetime first undergoes smooth polynomial expansion and then exponential inflation at late times. The bubble accelerates from zero velocity to the speed of light at late times. The same conclusion also holds for the bubble solution of eq. (3.1) which also undergoes inflation along the worldvolume directions. A quick way to see this is examine the constraint $-(Z^1)^2 + \sum_i (Z^i)^2 = r^2$. This constraint, along with the fact that the brane flattens out at infinity, shows that the bubble approaches the speed of light at late times. Finally, we note that of the two bubbles, the Witten bubble accelerates from zero velocity to the speed of light more slowly for $p = D$.

The surface of the Witten bubble undergoes contraction and then expansion. In fact, this surface of the bubble in eq. (3.18) has the metric of de Sitter space

$$r_0^2(-d\tau^2 + \cosh^2 \tau d\Omega^2_{D-2}) \, . \tag{3.19}$$

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The bubble solution is in effect a way to embed de Sitter space into an asymptotically flat background solution. Ref. [25] gave a discussion as to whether one can regard the Witten bubble to be a full solution of classical string theory. Having obtained a similar bubble solution one can ask the same question, do we have a string theory solution. Because this embedding satisfies the DBI equations, this solution naively captures the α' contributions in the small string coupling limit. However at our level of approximation we are neglecting possible higher order derivative corrections and the tachyon field. Since these solutions are not supersymmetric one would question if, by neglecting the tachyon field, one is being consistent. Although we have found new solutions to the DBI equations of motion, it is not apparent if these are also solutions of string theory.

Further properties of the Witten bubble can be found in [19, 21, 25].

3.4 Low dimensional bubbles

In analyzing the rest of the lower dimensional bubbles, we drop the χ direction. Then for the two brane case, $p = 3$, we find that the profile of the excitation Z^4 is not logarithmic and flattens out at infinity. This bubble solution should be relevant to the decay of a parallel two-brane and anti-brane pair. In contrast, the volcano solutions for the case of two-branes had logarithmic behavior at infinity. Although the volcano solution is a particular two brane configuration, it is not related to annihilation of parallel two-branes. When $p = 3$ we can also find another time dependent solution. This solution has the same spatial embedding as the other bubbles except it has an electric field in the angular ϕ direction

$$F_{\tau\phi} = \sqrt{-B^2 + r_0^4}\cosh\tau \tag{3.20}$$

which comes from analytically continuing the volcano solution with a magnetic field in eq. (2.15). In this case the χ direction does not seem to play a role and so could be dropped. We can therefore regard this too as a time dependent two-brane configuration.

For the case $p = 2$, we find a one dimensional time dependent solution which one can explicitly write down. When $p = 1$ we have a point particle and since there is no angular coordinate to analytically continue, it might appear that this configuration has no time evolution. Realizing, however, that we can also analytically continue the cartesian coordinates, we find solutions for a free particle. A discussion of the annihilation of zero-branes can be found in [22].

4. Spacelike brane

In this section we find further solutions of the DBI equations of motion. These solutions will be very different from the bubble and volcano solutions we have discussed so far and in fact they will be embedded in spaces with two times. Yet even with the appearance of two times, their worldvolume will have just one timelike direction and they will share similarities with known supergravity S-brane solutions [11, 13, 12, 14, 15].

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4.1 S-brane embedding

Analytically continuing the embedding of the volcano solution in eq. (2.6) by taking $t \to i\chi$, $\theta \to -i\theta$ and $r \to i\tau$ we obtain

$$Z^0 = \chi \tag{4.1}$$

$$Z^1 = \tau \cosh\theta \tag{4.2}$$

$$Z^i = \tau \sinh\theta n^i \qquad i = 2, \ldots, p \tag{4.3}$$

$$Z^{p+1} = \int \frac{B}{\sqrt{\tau^{2p-2} - \tau_0^{2p-2}}} d\tau \,. \tag{4.4}$$

Since we made a triple analytic continuation the embedding space is not Minkowski and its metric $G_{MN} = (1, -1, 1, \ldots, 1, -1)$ has two times. The induced metric on this solution is

$$ds^2 = d\chi^2 - \frac{1 + A^2/\tau^{2p-2}}{1 - (\tau_0/\tau)^{2p-2}} d\tau^2 + \tau^2[d\theta^2 + \sinh^2\theta d\Omega_{p-2}^2] \tag{4.5}$$

$$A^2 = B^2 - \tau_0^{2p-2} \tag{4.6}$$

and the field strength

$$F_{\chi\tau} = \frac{-A}{\sqrt{\tau^{2p-2} - \tau_0^{2p-2}}} \tag{4.7}$$

is an explicitly time dependent electric field. Here we must take $B^2 \geq \tau_0^{2p-2}$ just like for the volcano solution. Even though the embedding space has two times, this S-brane type solution is still well behaved in two respects. The DBI action is still real and it is possible to discuss time-evolution on the brane. The induced metric on this solution has at most only one independent time-like direction and if we stay in region $\tau \geq \tau_0$ then τ can be used to parametrize the worldvolume time. If we naively try to take $\tau \leq \tau_0$ then the worldvolume is spacelike. In this paper we will not discuss this spacelike region, or attempt to discuss any relationship between the two embedding times and the possible holographic reconstruction of a timelike direction proposed for spacelike branes.

For the case without charge, $A = 0$, the induced metric

$$ds^2 = d\chi^2 - \frac{d\tau^2}{1 - (\tau_0/\tau)^{2p-2}} + \tau^2[d\theta^2 + \sinh^2\theta d\Omega_{p-2}^2] \tag{4.8}$$

bears some similarity to the known S-brane supergravity solutions in $p+1$ dimensions. For example the metric of an S0-brane solution in four dimensions discussed in [12] is

$$ds_{S0}^2 = \frac{\tau_0^2}{Q^2}\left(1 - \left(\frac{\tau_0}{\tau}\right)^2\right) d\chi^2 - \frac{Q^2}{\tau_0^2}\frac{d\tau^2}{1 - (\tau_0/\tau)^2} + \frac{Q^2}{\tau_0^2}\tau^2[d\theta^2 + \sinh^2\theta d\phi^2] \,. \tag{4.9}$$

However due to the fact that the known S-branes contain a dilaton, a form field, or a field strength, there is actually another solution to Einstein's equations which is more like the DBI solution above. It is the triple analytic continuation of a $D+1$ Schwarzschild black hole

$$ds^2 = \left(1 - \left(\frac{\tau_0}{\tau}\right)^{D-2}\right) d\chi^2 - \frac{d\tau^2}{1 - (\tau_0/\tau)^{D-2}} + \tau^2[d\theta^2 + \sinh^2\theta d\Omega_{D-2}^2] \tag{4.10}$$

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where we have taken the analytic continuation $t \to i\chi$, $r \to \ i\tau$ and $\theta \to \ i\theta$ of the Schwarzschild solution. Since this metric is nothing but the analytic continuation of the Schwarzschild metric, it clearly satisfies the vacuum Einstein equations.

4.2 Properties

As mentioned in subsection 3.1, if the embedding space had only one time then this would imply that the embedded solution is globally hyperbolic. Since it turned out that our embedding space has two times, one then wonders if this was just an artifact of the embedding or if it not possible to embed this solution into a space with one time. If not, then there are some possible reasons why one time-like direction does not suffice. The first reason is that this spacelike brane seems to have a singularity at τ_0. This induced metric has singular components when $\tau = \tau_0$. It is known that singularities in the time direction can lead to spaces not being globally hyperbolic. The second reason is that spacelike branes are supposed to represent a finely tuned configuration for a system. Perhaps this fine tuning could mean that the system is correlated so it is not enough to specify the initial conditions of the system at one moment in time so there is no Cauchy surface. The size of the correlation effects could then give the S-branes a finite width.

This new solution does not appear to be a brane and anti-brane configuration but instead seems to be an unstable brane configuration. Let us use the worldvolume time to analyze the shape of this brane. It is possible to use the parameter τ to obtain sensible one time evolution along the worldvolume as long as we take $\tau \geq \tau_0$. In the above spherical coordinate system we find some interesting features of this solution. For comparison we first review the bubble case where the spatial directions satisfied the relationship $(Z^i)^2 = r^2\cosh^2\tau$. This shows us that at time $\tau = 0$ the solution starts at rest and that at late times the solution has accelerated to the speed of light. For the S-brane type solution we have $(Z^i)^2 = \tau^2\sinh^2\theta$. Yet since we also take $\tau \geq \tau_0$, the solution begins with some finite initial velocity proportional to τ_0. Further there is a gradient of initial velocities as we vary θ. The spatial origin which is located at $\theta = 0$ does not move while the points at large θ move very quickly.

It is also instructive to analyze the time evolution of this brane using the coordinate system

$$T = \tau\cosh\theta\,, \qquad W = \tau\sinh\theta \tag{4.11}$$

which is similar to the coordinate transformation for the bubble. In this case the induced metric becomes

$$ds^2_{\text{S-type}} = d\chi^2 - dT^2 + dW^2 - \frac{1 + A^2/\tau_0^{2p-2}}{(\sqrt{T^2 - W^2}/\tau_0)^{2p-2} - 1}\frac{(WdW - TdT)^2}{T^2 - W^2} + W^2 d\Omega^2_{p-2}\,. \tag{4.12}$$

In terms of the W, T coordinates, the restriction $\tau^2 \geq \tau_0^2$ becomes

$$W^2 \leq T^2 - \tau_0^2 \tag{4.13}$$

so we live in the ball bounded by a maximum sphere. Also the metric singularity at $\tau = \tau_0$ turns into a singularity when $W^2 = T^2 - \tau_0^2$. Along the spatial directions, Z^i with $i = 2, \ldots, p$, this brane can be said to represent an expanding (or contracting) brane with

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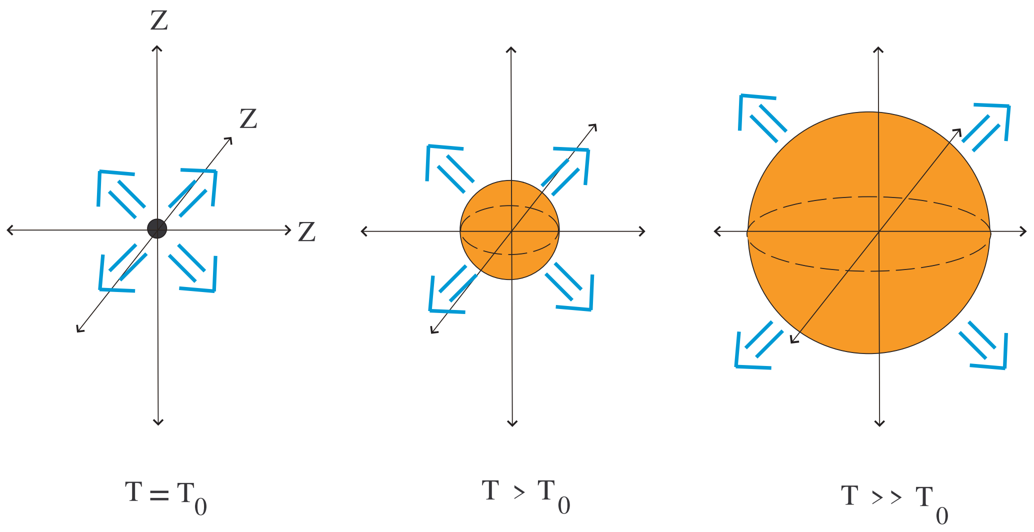

Figure 9: Along the spatial directions, the S-brane type solution begins at a point and expands outwards.

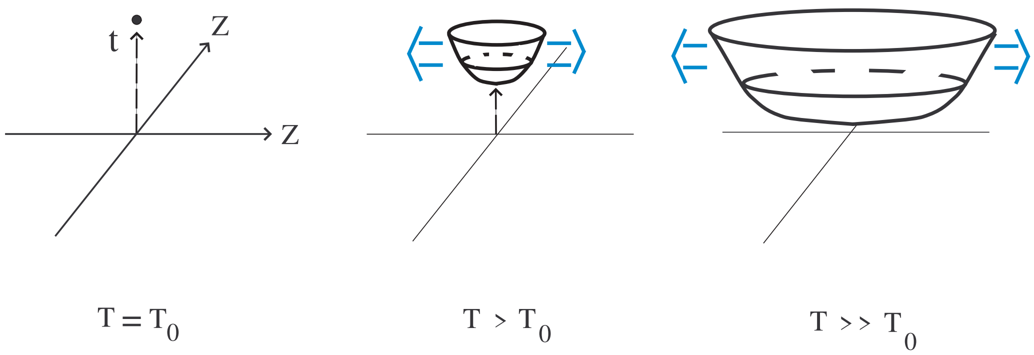

Figure 10: In the full embedded space, the S-brane type solution begins highly peaked in a timelike direction and relaxes into a flat configuration.

the condition that the hole always begins with zero size since τ and hence T starts at τ_0. The time evolution is shown schematically in figure 9. Although the brane does not start off with an infinite velocity, it expands to infinite size. At late times in this coordinate system, the brane becomes essentially flat Minkowski space. If we include the excitation direction, this brane has the form of a peaked excitation which eventually flattens out as shown in figure 10. In the two previous cases, it was shown that we could glue together solutions to form a brane and anti-brane system. Although in this case since the excitation direction is timelike, since there is only one worldvolume time it seems possible also to perform the gluing procedure. From the perspective of the brane observers the excitation direction serves only to time dilate events. The worldvolume then becomes topologically a double cover of a ball and instead of going to the edge of the ball we arrive on an identical second surface.

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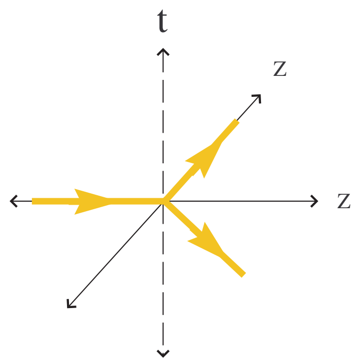

Figure 11: Tachyon interaction process

4.3 Low dimensional solutions

For the case $p = 3$, we can find another spacelike brane with time dependent electric field

$$F_{\theta\phi} = \sqrt{B^2 - r_0^4}\sinh\theta \tag{4.14}$$

This is the analytic continuation of the volcano solution with a magnetic field.

In treating the other solutions of low dimensionality, take $\chi = 0$ and $\tau > \tau_0$. For the case $p = 2$, take a constant timelike slice of τ. Then in this case $(Z^1)^2 - (Z^2)^2$ is constant and the span of θ gives half of a hyperbola at a constant Z^3 height. As we evolve to more positive τ times, the brane moves away from the origin and decreases in Z^3 height. Due to the low dimensionality of this solution it is also possible to reinterpret the embedding space with metric $(-1, 1, -1)$ as having only one time. This solution has the same form as the time dependent $p = 2$ bubble solution. When p is greater than two, then it is not possible to reinterpret the embedding space as having one time.

For the case $p = 1$, the worldvolume is zero dimensional, and we are describing a point like object. We find a freely moving object which is purely in the time direction since the embedding space has metric $(-1, -1)$. However we can again interpret this as a purely spatial trajectory and add an extra embedding dimension t of plus signature. The embedding space now has the metric $(-1, -1, 1)$. In this case the dynamics of this object are equivalent to those of a freely propagating tachyon. These spacelike branes are tachyons with spacelike trajectories which exist only at one moment of time t in the embedding space. It is interesting to explore to what extent string junctions can be interpreted as a tachyon interaction process as shown in figure 11.

Further properties of these solutions will be discussed elsewhere.

5. Conclusions

We have provided time dependent solutions to the DBI equation of motions. These solutions are similar to Witten's bubble of nothing, de Sitter space and to S-branes and one is tempted to consider them as "dual" of the closed string gravity solutions. The interpretation of the S-brane type solution is not very clear although it seems to be a generalization

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of a tachyon. It is an unstable brane configuration with singularities, and its dynamics take place in a background space with two times. Although the interpretation of two times is not fully understood, the worldvolume has only one timelike direction which we used for time evolution.

The bubble solution has a clear interpretation as being a time dependent brane and anti-brane annihilation process that can be solved exactly and which resembles some cosmological scenarios. For example one could imagine that we live in an inflationary universe due to the annihilation of a $D3 - \overline{D3}$ pair. A problem immediately arises since this space has eternal inflation. However since more than one bubble can nucleate, the interactions of different bubbles could potentially serve as a qualitative reason why inflation should end. Another problem with a such a scenario however is how one could account for the fact that such branes actually got close enough for a wormhole to have a reasonable chance to form. Taking into account the mutual attraction between the branes could potentially solve this problem.

To increase the chances for wormholes to nucleate, one could increase the dimensionality of the branes. For example we could take a $D4 - \overline{D4}$ pair which is much more likely to intersect. If the branes intersect then they will be close enough in general for wormholes to form with non-negligible probability. In this case one wonders if it is possible to have our observable universe located on the bubble which is four dimensional de Sitter space with matter accumulating near the bubble as it sweeps throughout the brane. One can also examine the case of fivebranes [8] and look for a new decay process. In this case one would look for a vortex solution which would be our three brane universe.

In this paper we have discussed the open string tachyon from the DBI point of view. Of course one wonders about the closed string tachyons. In general it appears to be a difficult question to imagine how to give a formulation where both instabilities are treated concurrently. It is possible however to make some qualitative statements about known instabilities and see if they agree with expectations. For example in a set up similar to Fabinger and Horava but for p-branes instead of E_8 boundaries, one can calculate that the orbifold projection of the Witten bubble has a nucleation probability proportional to $\exp(-L^{p-2}/g^2 l_s^{p-2})$ where L is the separation distance between branes. In the case of the DBI bubble we find that the nucleation rate is proportional to $\exp(-L^{p+1}/g l_s^{p+1})$, which has the same form as the decay rate calculated [22] using the tachyonic DBI action. In the case of very large separation the Witten bubble is the more dominant decay process. In the case where the brane separation is of the order of the string scale, the Witten bubble dominates for large string coupling while the DBI bubble dominates for small string coupling. The expectation is that for light branes, the backreaction is small and so the open string tachyon should dominate, while for heavy branes the backreaction is large and so the closed string tachyon should dominate.

For small string coupling the above results seem to lead to the following possible picture of the tachyon decay process for a brane and anti-brane. A brane and anti-brane at large separation create Witten bubbles between them and approach each other. The bulk of spacetime is annihilating. When the brane separation approaches the string scale, the other bubbles take over and we proceed with the safer decay process where just the brane

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and anti-brane annihilate. Although both tachyons are present, the safer one takes over and saves the bulk of the universe while annihilating the branes. In this account we are neglecting the interaction of the various bubbles which is expected to be complicated. An explicit solution describing two Witten bubbles has recently been found [26].

Given the above qualitative statements, one can also attempt to make some contact with the homogenous tachyon decay process discussed by Sen. Although the bubble decay process is inhomogeneous, in the limit of small string coupling and brane separation, the probability of bubble production becomes very large. In general one would then expect to see an almost uniform distribution of bubble type solutions when the branes are nearly coincident.

Finally it would be interesting to see if solutions with time dependent field strength can give a physical picture for the problem of the U(1) gauge field [27]–[32]. New explicit time dependent solutions describing the fate of the electric field can be found in [33, 34] and will also play a role in a future paper.

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